M.Sc. - I (Mathematics) (New CBCS Pattern) Semester-I

PSCMTH03 - Paper-III : Topology-I

P. Pages : 2 **GUG/S/25/13739**

Time : Three Hours Max. Marks : 100

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Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that the union of a denumerable number of denumerable sets is a denumerable set. **10**

b) Prove that the set of all rational number is denumerable. **10**

OR

c) Prove that $2^a > a$ for every cardinal number a **10**

d) Show that the set of all real numbers is uncountable. **10**

UNIT – II

2. a) Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ show that derived set of {a} & {c} are {c} & ϕ respectively. **10**

b) Prove that F is closed set iff. F^C is an open set. **10**

OR

c) Prove that $i(E) = \left[C\left(E^C\right)\right]^C$ **10**

d) Prove that for any set E in a topological space $C(E) = E \cup d(E)$ **10**

UNIT – III

3. a) If F is a continuous mapping of (x, τ) into (x^*, τ^*), then prove that f maps every compact subset of X onto a compact subset of X*. **10**

b) If C is connected subset of a topological space (x, τ) which has a separation $X = A \mid B$ then prove that $C \subseteq A$ or $C \subseteq B$ **10**

OR

c) If F is a Homeomorphism of X onto X*, then prove that F maps every isolated subset' of X onto an isolated subset of X*. **10**

d) Prove that compact subset of a topological space is countably compact. **10**

UNIT – IV

4. a) Prove that a topological space X is a To-space iff the closures of distinct points are distinct. **10**

b) Let X be a T_1 - space & $E \subseteq X$, then prove that x is the limit point of E iff every open set containing x contains infinite number of distinct points of E. **10**

OR

c) Prove that every combact Hausdorff space is normal (T_4) **10**

d) Give an example of a topological space which in T_1 but not T_2 space. **10**

5. a) Prove that the set of integers Z is denumerable. **5**

b) Prove that if E is open iff E = i (E) **5**

c) Prove that closure of a connected set is connected. **5**

d) Define T_0, T_1 & T_2 space. **5**

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